Free falling from infinity to a black hole

Problem: You need inertial reference frames to agree on the energy of the object free falling in. No inertial reference frame, no matter how small, will be valid for the entire trip up to the event horizon.

The Equivalence principle claims that locally, we are unable to distinguish from uniform acceleration and that of being in a gravitational field.

This then tells me that we need to limit the life of our local free float frame. This is okay and a valid thing to do.

Who does the test for inertial reference frames? Just the owner or everyone?

* If these frames are local, the test must be done locally?

Pages 75 and 76 of Spacetime Geometry: We examine the physical situation using flat spacetime, so the relationship is described that way (velocity at time t) and its generalized to a coordinate independent form (Tensors) and we can then transfer it back to curved spacetime?